1. Hausübung, **Statistische Physik**

abzugeben am Donnerstag, 20.10.2011

Aufgabe H1 *Law of large numbers* (6 Punkte)

Suppose one throws an unbiased coin N times. We expect that if N is large, we are likely to obtain roughly as many heads as tails. Let's make this precise.

a. For large N, estimate the probability of obtaining $\frac{1}{2}N$ heads.

b. For large N, show that the probability that the number of head falls between $\frac{1}{2}N - \sqrt{N}$ and $\frac{1}{2}N + \sqrt{N}$ is independant of N.

Note: you can make use of Stirling's approximation, and you are allowed to refer to results obtained in the course.

Aufgabe H2 *Ideal gas* (6 Punkte)

Consider N distinguishable free particles, each characterized by three integer momentum components $(k_1, k_2, k_3) \in \mathbb{Z}^3$ and kinetic energy proportional to $\sum_i k_i^2$.

a. Show that for large U and large N, the number of states with total energy between U and $U + \epsilon$, for $\epsilon \ll U$, has the form

$$g(N, U) \simeq \epsilon f(N) U^{3N/2}$$

for some function $f(N)$.

Hint: Consider first the number of states with energy *less* than U and take its derivative with respect to U. The volume of an n-dimensional sphere of radius r is proportional to r^n.

b. Consider two such systems with N_1 and N_2 particles respectively in thermal contact: i.e. assuming that all configurations with total energy between U and $U + \epsilon$ are equally likely. What is the most likely value of the energy U_1 of the system with N_1 particles?

Hint: the form of the function $f(N)$ is not needed to answer this question.